Suppose g and h are differentiable at a. Then, prove g/h is differentiable at a, and

(g/h)'(a)= g'(a)h(a)-g(a)h'(a)/ $(h(a))^2$.

Suppose $f(x) = g(x)/h(x)$, where $h(x) \neq 0$ and $g$ and $h$ are differentiable.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{g(x+\Delta x)}{h(x+\Delta x)} - \frac{g(x)}{h(x)}}{\Delta x}$$

We pull out the $1/\Delta x$ and combine the fractions in the numerator:

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left( \frac{g(x+\Delta x)h(x) - g(x)h(x+\Delta x)}{h(x)h(x+\Delta x)} \right)$$

Adding and subtracting $g(x)h(x)$in the numerator:

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left( \frac{g(x+\Delta x)h(x) - g(x)h(x) - g(x)h(x+\Delta x) + g(x)h(x)}{h(x)h(x+\Delta x)} \right)$$

We factor this and multiply the $1/\Delta x$ through the numerator:

$$= \lim_{\Delta x \to 0} \frac{\frac{g(x+\Delta x) - g(x)}{\Delta x} h(x) - g(x) \frac{h(x+\Delta x) - h(x)}{\Delta x}}{h(x)h(x+\Delta x)}$$

Now we move the limit through:

$$= \frac{\lim_{\Delta x \to 0} \left( \frac{g(x+\Delta x) - g(x)}{\Delta x} \right) h(x) - g(x) \lim_{\Delta x \to 0} \left( \frac{h(x+\Delta x) - h(x)}{\Delta x} \right)}{h(x) \lim_{\Delta x \to 0} h(x+\Delta x)}$$

By the definition of the derivative, the limits of difference quotients in the numerator are derivatives. The limit in the denominator is *h*(*x*) because differentiable functions are continuous. Thus we get:

$$= \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}.$$