Answer on Question #82487 – Math – Real Analysis

Question

1) Let $x_1 = 8$ and $x_{n+1} = 2 + x_n/2$ for all n belongs to $\boldsymbol{N}$. Show that X is bounded and monotone. Also find the limit.

Solution

1) If $x > 4$ then $2 + \frac{x}{2} > 4$, so $x_n > 4$ (bounded from below by 4)

If $x_n > 4 \Longrightarrow \frac{x_n}{2} > 2 \Longrightarrow x_n - \frac{x_n}{2} > 2 \Longrightarrow x_n > 2 + \frac{x_n}{2} \Longrightarrow x_n > x_{n+1}$

So x_n is monotone decreasing and bounded by 4.

Then it has some limit, say: $a = \lim\limits_{n\to\infty} x_n$

Then $a = \lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = 2 + \frac{a}{2}$, so $a = 2 + a/2$.

So $a = 4$.

Question

2) Let $x_1 > 1$ and $x_{n+1} = 2 - 1/x_n$ for all n belongs to $\boldsymbol{N}$. Show that $\{x_n\}$ is bounded and monotone. Find the limits.

Solution

If $x > 1 \Longrightarrow \frac{1}{x} < 1 \Longrightarrow -\frac{1}{x} > -1 \Longrightarrow 2 - \frac{1}{x} > 1$

So if $x_1 > 1$ then every $x_n > 1$

Let's prove, that for every $x > 0 \Longrightarrow x > 2 - 1/x$

$(x-1)^2 > 0$, so $x^2 > 2x - 1$, so $x > 2 - 1/x$

(we could divide both parts by x, because $x > 0$)

so $x_{n+1} < 2 - 1/x_n$. So x_n is monotone decreasing and bounded by 1 from below.

Then it has some limit, say: $a = \lim\limits_{n\to\infty} x_n$

Then $a = \lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = 2 - \frac{1}{a}$

So (a = 2 − 1/a), so $a = 1$.

Answer provided by https://www.AssignmentExpert.com