1. Establish either the sequence $X = x_n$ converges or diverges where

Question

i)

$$x_n = \frac{n}{n+1}$$

Solution

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

Answer: the sequence converges

Question

ii)

$$x_n = \frac{(-1)^n n}{n+1}$$

Solution

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{n}{n+1} = 1 \quad for\ even\ n$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{-n}{n+1} = -1 \quad for\ odd\ n$$

So, the sequence has not limit.

Answer: the sequence diverges

Question

iii)

$$x_n = \frac{n^2}{n+1}$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{n^2}{n+1} = \lim_{n\to\infty}\left(n - 1 + \frac{1}{n+1}\right) = \infty$$

Answer: the sequence diverges

2. Find the limits of the sequences

Question

i)

$$\left(2 + \frac{1}{n}\right)^2$$

Solution

$$\lim_{n\to\infty}\left(2 + \frac{1}{n}\right)^2 = 2^2 = 4$$

Answer: 4.

Question

ii)

$$\frac{(-1)^n}{n+2}$$

Solution

$$\lim_{n\to\infty}\frac{(-1)^n}{n+2} = 0$$

Answer: 0.

Question

iii)

$$\frac{n^{1/2} - 1}{n^{1/2} + 1}$$

Solution

$$\lim_{n\to\infty} \frac{n^{1/2}-1}{n^{1/2}+1} = \lim_{n\to\infty} \frac{n^{1/2}(1-1/n^{1/2})}{n^{1/2}(1+1/n^{1/2})} = \frac{1-0}{1+0} = 1.$$

Answer: 1.

Question

iv)

$$\frac{n+1}{nn^{1/2}}$$

Solution

$$\lim_{n\to\infty} \frac{n+1}{nn^{1/2}} = \lim_{n\to\infty} \frac{1+1/n}{n^{1/2}} = 0$$

Answer: 0.

Question

v)

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} \ for \ 0 < a < b$$

Solution

$$\lim_{n\to\infty} \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{b^{n+1}}+\frac{b^{n+1}}{b^{n+1}}}{\frac{a^n}{b^{n+1}}+\frac{b^n}{b^{n+1}}} = \frac{1}{\frac{1}{b}} = b.$$

Answer: b.