Answer on Question #82483 – Math – Differential Equations

Question

Solve the following ODE using the power series method:

$$(x+2)y'' + xy' - y = 0$$

Solution

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$$

$$(x+2)\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + x\sum_{n=1}^{\infty} a_n n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n-1} + 2\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=1}^{\infty} a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$n-2 \to N, n \to N+2$

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n-1} + 2\sum_{N+2=2}^{\infty} a_{N+2}(N+2)(N+2-1)x^{N+2-2} +$$

$$+\sum_{n=1}^{\infty} a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$n-1 \to N, n \to N+1$

$$\sum_{N+1=2}^{\infty} a_{N+1}(N+1)(N+1-1)x^{N+1-1} + 2\sum_{N=0}^{\infty} a_{N+2}(N+2)(N+1)x^N +$$

$$+\sum_{n=1}^{\infty} a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{N=1}^{\infty} a_{N+1}(N+1)N x^N + 2\sum_{N=0}^{\infty} a_{N+2}(N+2)(N+1)x^N +$$

$$+\sum_{N=1}^{\infty} a_N N x^N - \sum_{N=0}^{\infty} a_N x^N = 0$$

$$\sum_{N=1}^{\infty}(a_{N+1}(N+1)N+2a_{N+2}(N+2)(N+1)+a_N N-a_N)x^N+$$
$$+2a_{0+2}(0+2)(0+1)-a_0=0$$

$$4a_2-a_0=0$$
$$a_{N+1}(N+1)N+2a_{N+2}(N+2)(N+1)+a_N(N-1)=0$$

$$y=a_0+\sum_{n=1}^{\infty}a_n x^n$$
$$4a_2-a_0=0$$
$$a_{n+2}=-\frac{a_{n+1}(n+1)n+a_n(n-1)}{2(n+2)(n+1)}\ ,n\geq 1$$