

Answer on Question #82410 – Math – Real Analysis

Question

Show that $8^{1/2}$ is not an integer.

Solution

The set of integers, denoted by \mathbb{Z} , is formally defined as follows:

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

Integers include natural numbers (counting numbers), the opposites of the natural numbers and zero.

$$4^{1/2} = 2, 2 > 0, \text{ since } 2^2 = 4$$

$$9^{1/2} = 3, 3 > 0, \text{ since } 3^2 = 9$$

$$0 < 4 < 8 < 9 \Rightarrow 4^{1/2} < 8^{1/2} < 9^{1/2}$$

$$\text{Then } 2 < 8^{1/2} < 3$$

There is no integer number between 2 and 3.

Thus, $8^{1/2}$ is not an integer.