Answer to Question #82335, Math / Real Analysis

Question

(1) For a, b belongs to R

(i)$-(-a) = a$

(ii)$-1(a) = -a$

(iii) $-(a+b) = (-a) + (-b)$

(iv) $-(a)(-b) = ab$

(2) Find real number x such that $|x+1| + |x+2| < 7$

Solution

(1) Given: $a, b \,\epsilon R$

(i) Since $-a$ is additive inverse of a it satisfies $-a + a = a + (-a) = 0$.That is a is an additive inverse of –a.

Since $-(-a)$ is additive inverse of $-a$ it satisfies

$-a + \left(-(-a)\right) = -(-a) + (-a) = 0.$

As we know that additive inverse is unique, we get $a = -(-a)$

Hence, proved

(ii) To prove for every $a\epsilon R$ the additive inverse of a is the product of additive inverse of 1 and a , it is sufficient to prove that $(-1)a + a = a + (-1)a = 0$

We will show the second equality holds

$a + (-1)a = 1 \cdot a + (-1) \cdot a$ by multiplicative identity

$= (1 + (-1)) \cdot a$ by distributivity

$= 0 \cdot a$ by additive inverse

$= 0$ for every $a\epsilon R$,$0 \cdot a = a \cdot 0 = 0$

So both equalities hold and as $(-1)a$ and $-a$ are both additive inverse of a, $-1(a) = -a$

Hence, proved

(iii) Since – $(a+b)$ is an additive inverse of $a+b$. We have – $(a+b)+(a+b)=(a+b)+-(a+b)$

We also have

$(a+b)+((-a)+(-b))=((-a)+(-b))+(a+b)$ by commutativity of addition

$=(-a)+((-b)+a)+b$ by associativity of addition

$=(-a)+(a+(-b))+b$ by commutativity of addition

$=((-a)+a)+((-b)+b)$ by associativity of addition

$=0+0$ by additive inverse of a and b

$=0$ by additive identity

Hence, $(-a)+(-b)$ is also additive inverse of $a+b$.

By uniqueness of additive inverse we get $-(a+b)=(-a)+(-b)$

Hence, proved

(iv)

$(-a)(-b)=(-1(a))(-1(b))$ $-1(a)=-a$

$=(-1)(a)(-1)(b)$ by commutativity of multiplication

$=(-1)(-1)(a)(b)$ by associativity of multiplication

$=(a)(b)$ $(-1)(-1)=1$

Hence, proved

(2) $|x+1|+|x-2|<7$

Here we have two check points -1 and 2.

Consider below three cases

Case (i): If $x<-1$ then $|x+1|+|x-2|$ expands as $-x-1-x+2<7$ gives $x>-3$

Hence, we get $-3<x<-1$

Case (ii): If $-1\leq x<2$ then $|x+1|+|x-2|$ expands as $x+1-x+2<7$ gives $3<7$ which is true for all$x\epsilon R$. Hence, for $-1\leq x<2$ given inequality holds true.

Case (iii): If $x\geq 2$ then $|x+1|+|x-2|$ expands as $x+1+x-2<7$ gives $x<4$

Hence, we get $2\leq x<4$

combine all three ranges we get

$(-3 < x < -1)\ or\ (-1 \leq x < 2\ and\ true\ \forall\ x\epsilon R)\ or\ (2 \leq x < 4)$

That is $-3 < x < 4$

Hence, $-3 < x < 4\ or\ x\epsilon(-3,4)$

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