Answer on Question #82286 – Math – Differential Equations

Question

Find a power series solution in powers of $x-1$ of the initial-value problem.

$$xy''(x)+y'(x)+2y(x)=0, \qquad y(1)=2, y'(1)=4$$

Solution

$$y(x)=y(1)+y'(1)(x-1)+\frac{y''(1)}{2!}(x-1)^2+\frac{y'''(1)}{3!}(x-1)^3+\frac{y^4(1)}{4!}(x-1)^4+\cdots$$

$$y''(x)=-\frac{y'(x)+2y(x)}{x}$$

$$y''(1)=-(4+2\cdot 2)=-8$$

$$y'''(x)=-\frac{x(y''(x)+2y'(x))-y'(x)-2y(x)}{x^2}$$

$$y'''(1)=8-2\cdot 4+4+2\cdot 2=8$$

$$y^{(4)}(x)=-\frac{x(y'''(x)+2y''(x))-y''(x)-2y'(x)}{x^2}+\frac{x^2(y''(x)+2y'(x))-2x(y'(x)+2y(x))}{x^4}$$

$$y^{(4)}(1)=-(8-2\cdot 8)-8+2\cdot 4-8+2\cdot 4-2\cdot(4+2\cdot 2)=-8$$

$$y(x)=2+4(x-1)-\frac{8}{2!}(x-1)^2+\frac{8}{3!}(x-1)^3-\frac{8}{4!}(x-1)^4+\cdots$$

$$y(x)=2+4(x-1)-4(x-1)^2+\frac{4}{3}(x-1)^3-\frac{1}{3}(x-1)^4+\cdots$$

Answer:

$$y(x)=2+4(x-1)-4(x-1)^2+\frac{4}{3}(x-1)^3-\frac{1}{3}(x-1)^4+\cdots$$