Answer on Question #82284 – Math – Differential Equations

Question

$$(x^2 - 1)y''(x) + 3xy'(x) + xy(x) = 0, \qquad y(0) = 4, y'(0) = 6$$

Solution

The given equation cannot be solved in standard functions.

We can represent $y(x)$ as Maclaurin series:

$$y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \cdots$$

Then:

$$y''(x) = -\frac{3xy'(x) + xy(x)}{x^2 - 1}$$

$$y''(0) = 0$$

$$y'''(x) = -\frac{(3y'(x) + 3xy''(x) + y(x) + xy'(x))(x^2 - 1) - (2x - 1)(3xy'(x) + xy(x))}{(x^2 - 1)^2}$$

$$y'''(0) = 3 \cdot 6 + 4 = 22$$

Answer:

$$y(x) = 4 + 6x + \frac{11}{3}x^3 + \cdots$$