Answer on Question #82242 – Math – Functional Analysis

Question

show that the addition of the type dg/dx to the integrand function leaves the euler equation in the same form

Solution

We consider the functional

$$J[y] = \int_{x_0}^{x_1} f(x, y, y')dx.$$

Suppose $J[y]$ has an extremum for the curve y. Then we have $\delta J[y] = 0$, and therefore, the Euler-Lagrange equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0.$$

Now let's take a new function

$$F(x, y, y') = f(x, y, y') + \frac{dg(x, y)}{dx}.$$

Note, that $g(x, y)$ must be function of only (x, y), and not of y'. The new functional will be

$$J_1[y] = \int_{x_0}^{x_1} F(x, y, y')dx = \int_{x_0}^{x_1} f(x, y, y')dx + \int_{x_0}^{x_1} \frac{dg(x, y)}{dx}dx = \int_{x_0}^{x_1} f(x, y, y')dx + \int_{x_0}^{x_1} dg(x, y)$$

$$= \int_{x_0}^{x_1} f(x, y, y')dx + g\big(x_0, y(x_0)\big) - g\big(x_1, y(x_1)\big).$$

But $g\big(x_0, y(x_0)\big)$ and $g\big(x_1, y(x_1)\big)$ are constants, and their variations are null. Therefore, we have

$$\delta J_1[y] = \delta \int_{x_0}^{x_1} F(x, y, y')dx = \delta \int_{x_0}^{x_1} f(x, y, y')dx = \delta J[y].$$

That means

$$\delta J[y] = 0 \ \Rightarrow \ \delta J_1[y] = 0$$

and so the new Euler-Lagrange equations are

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0.$$

As we can see this addition to the integrand function indeed leaves the Euler-Lagrange equation in the same form.

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