Answer on Question #82224 – Math – Geometry

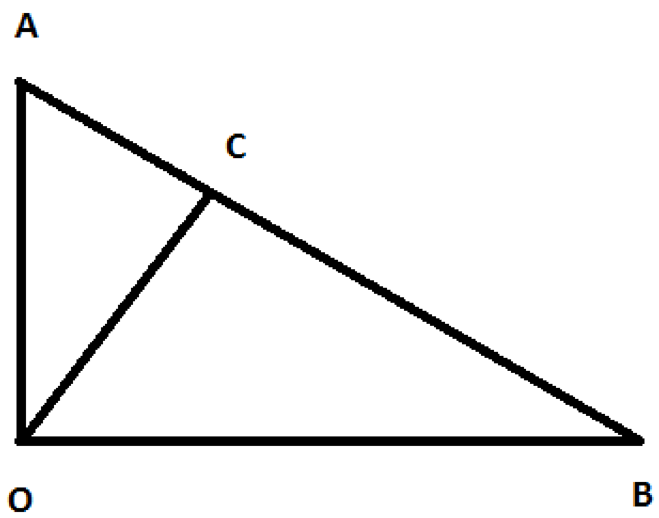
Question

1. In ∆OAB, ∠AOB=90º. Let C be the point on the segment AB such that OC⊥−AB.

Prove

|CA|/|CB| = |OA|^2/|OB|^2

Solution



According to the triangles similarity theorem: If two angles of one triangle are equal to two angles of another, then the triangles are similar. ∆OAB, ∆OAC, ∆OBC – similar.

∠AOB=∠OCA=∠OCB=90º, ∠OBA=∠OBC=∠AOC= α and ∠OAB=∠OAC=∠COB=90º – α.

So can write:

$\tan\alpha = \frac{|OA|}{|OB|} = \frac{|CA|}{|OC|} = \frac{|OC|}{|CB|}$ (1),

$\sin^2\alpha + \cos^2\alpha = 1$ (2),

$\frac{|CA|^2}{|OA|^2} + \frac{|CB|^2}{|OB|^2} = 1,$

$\frac{|CA|^2}{|OA|^2} = 1 - \frac{|CB|^2}{|OB|^2} = \frac{|OB|^2 - |CB|^2}{|OB|^2},$

$\frac{|CA|^2}{|OB|^2 - |CB|^2} = \frac{|OA|^2}{|OB|^2},$

where $|OB|^2 - |CB|^2 = |OC|^2$, so have

$\frac{|CA|^2}{|OC|^2} = \frac{|OA|^2}{|OB|^2},$

from (1) obtain $|OC|^2 = |CA| \cdot |CB|$.

As a result,

$$\frac{|CA|^2}{|CA|\cdot|CB|} = \frac{|CA|}{|CB|} = \frac{|OA|^2}{|OB|^2}.$$