1. Find the number of positive and negative roots of the polynomial $P(x) = x^3 - 3x^2 + 4x - 5$ Find P(2) and P'(2) using synthetic division method.

Answer:

As per Descartes's rule of signs, the number of positive roots is equal to change in sign of$P(x)$, or is less than that by an even number.

Given polynomial is $P(x) = x^3 - 3x^2 + 4x - 5$

The signs of coefficients are in the form $+ - + -$

Since there are 3 changes of sign, there are 3 or 1 positive real roots

Similarly, for the number of negative real roots we have to find $P(-x)$and we have to count again

$P(-x) = -x^3 - 3x^2 - 4x - 5$

(It is same as reversing the signs on terms of odd degree)

The signs of coefficients are in the form $- - - -$

Since there is no change of sign, there are zero negative real roots.

As the total number of zeros of $P(x)$ is 3, that means it has 0 or 2 non-real Complex zeroes.

To evaluate $P(2)$

Using factor-remainder theorem we know that if a polynomial $P(x)$ is divides by $x - a$, the remainder is $P(a)$.

So the value of $P(2)$ by synthetic division as below

2	1	-3	4	-5
		2	-2	4
	1	-1	2	-1

Hence,
$P(2) = -1$

To evaluate $P'(2)$

$P'(x) = 3x^2 - 6x + 4$

2	3	-6	4
		6	0
	3	0	4

Hence,
$P'(2) = 4$