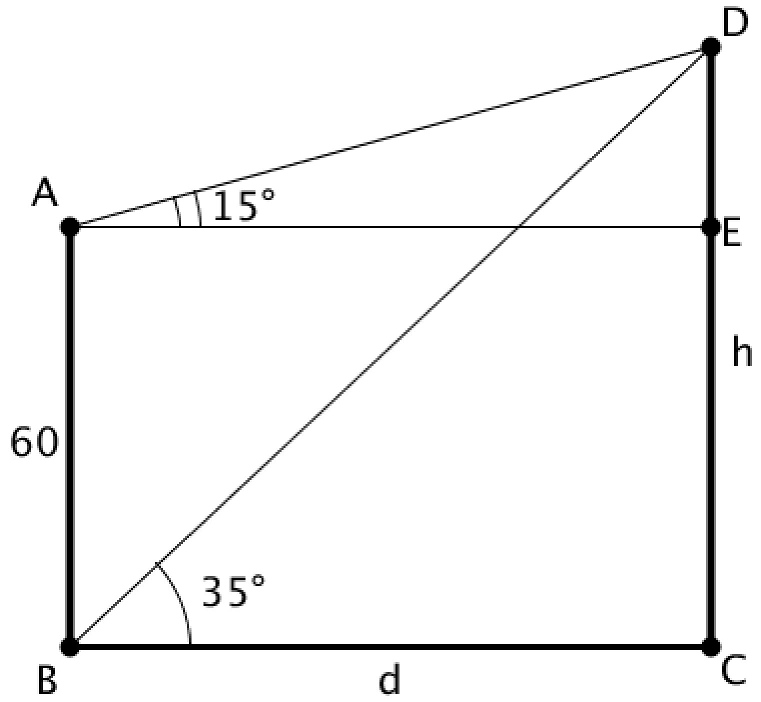
Answer on Question #82140 – Math – Trigonometry

Question

From the top of the building 60 m high, the angle of elevation of the top of a vertical pole is 15. At the bottom of the building the angle of elevation of the top of the pole is 35. Find (a) the height of the pole and (b) the distance of the pole from the building.



Solution

Assume that our building and pole is not a tower of Pisa (building and pole are perpendicular to the ground). Point "A" represents the top of building; point, "B" represents the bottom of building; "C" represents the bottom of pole; "D" represents the top of the pole. Line AB is the height of building (60 m); BC is the distance from building to the pole (let it be "d"); DC is the height of the pole (let it be "h"). Angle DBC is 35˚. If we draw a line parallel to the ground (BC) from the top of the building (point A), we get a projection of building on the pole. This line is AE. Thus, the quadrilateral ABCE is rectangle (AB=EC, AE=BC, all angles are 90˚). Using the condition of the question it means that angle DAE is 15˚.

Look at the triangle DBC, $tan(\angle DBC) = \frac{DC}{BC}$. In our notations it is $tan(35^{\circ}) = \frac{h}{d}$.

Now look at the triangle ADE. AE=BC=d, DE=h-60 => $tan(15^{\circ}) = \frac{h-60}{d}$.

Dividing both formulas gives: $\frac{tan(15^{\circ})}{tan(35^{\circ})} = \frac{h-60}{d} \cdot \frac{d}{h} = \frac{h-60}{h} = 1 - \frac{60}{h}$.

Now we can find an equation for h: $\frac{60}{h} = 1 - \frac{tan(15^{\circ})}{tan(35^{\circ})} \rightarrow h = \frac{60}{1-\frac{tan(15^{\circ})}{tan(35^{\circ})}}$.

From the table or using a calculator we have values of tangents:

$tan(15^\circ) \approx 0.268; \ \ tan(35^\circ) \approx 0.7; \ \ \frac{tan(15^\circ)}{tan(35^\circ)} \approx 0.383.$

$$h = \frac{60}{1 - 0.383} \approx \frac{60}{0.617} \approx 97.24$$

$$d = \frac{h}{tan(35^\circ)} \to d \approx \frac{97.24}{0.7} \approx 138.91$$

Answer: the height of pole is approximately 97.24 m and the distance to the pole is approximately 138.91 m.