

Answer on Question #82066 – Math – Geometry

Question

Let O be the origin of coordinate plane, A, B lie on the upper half plane satisfying $OA=OB$. If line OA has slope 1, line OB has slope -7 , what is the slope of line AB ?

Solution

Let A have coordinates (x, y) . $y = x$ because line OA has slope 1 which equals y/x .

Let B have coordinates $(-a, b)$. $b = 7a$ because line OB has slope -7 which equals b/a (line placed in left upper half plane)

By using the Pythagorean theorem calculating length of line OA :

$$\sqrt{x^2 + x^2} = x * \sqrt{2}$$

We know that the line OB have the same length so we can set correspondence between x and a :

$$\text{Length } OB \text{ equals } \sqrt{(7a)^2 + a^2} = x * \sqrt{2} \Leftrightarrow (7a)^2 + a^2 = 2 * x^2 \Leftrightarrow$$

$$\Leftrightarrow 25 * a^2 = x^2 \Leftrightarrow 5a = x$$

So what we've got? By using this correspondence we know that coordinates of A are $(5a, 5a)$,

Coordinates of B are $(-a, 7a)$. Let's enter a point C with coordinates $(-a, 5a)$. Line AC parallel to the x -axis because points A and C have the same ordinates. That means that the tangent of angle BAC equals minus slope of line AB because angle BAC adjacent to the angle which defines the slope of line AB ($180 - \text{angle } BAC$) so as we know $\tan(180 - \alpha) = -\tan(\alpha)$

Tangent of angle BAC equals $BC / AC = (7a - 5a) / (5a - (-a)) = 2a / 6a = 1 / 3$ hence slope of line AB equals $-1 / 3$

Answer: $-1 / 3$.