

Question 1. Show that $f = o(1)$ as $x \rightarrow a$ if and only if $f(x) \rightarrow 0$ as $x \rightarrow a$.

Solution. Recall that $f(x) = o(g(x))$ as $x \rightarrow a$ iff for any $\varepsilon > 0$ there is $\delta > 0$, such that $|f(x)| \leq \varepsilon|g(x)|$ for all x with $0 < |x - a| < \delta$. Use this definition in the case when $g(x) \equiv 1$ and obtain that for any $\varepsilon > 0$ there is $\delta > 0$ such that $|f(x)| \leq \varepsilon$ for all x , such that $0 < |x - a| < \delta$. By definition of the limit of a function this is the same as $\lim_{x \rightarrow a} f(x) = 0$. \square