

Answer on Question #81986 - Math - Combinatorics | Number Theory

Find the number of terms free from the radical sign in $\{(7)^{1/3} + (11)^{1/9}\}^{654}$.

According to the binomial theorem, it is possible to expand any power of $x + y$ into a sum of the form

$$(x + y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n.$$

Let $x = (7)^{1/3}$, $y = (11)^{1/9}$.

As $6+5+4=15$ the number 654 is divisible by 3 but is not divisible by 9. Free of radical terms will be the terms with power of y divisible by 9 including 0. The corresponding power of x will be divisible by 3. The 648 is divisible by 9, $648/9=72$, $648 + 9 = 657 > 654$. Thus we have 72 integers from 1 to 654 which are divisible by 9. As we must include 0. The answer will be $72+1=73$.