Answer on Question #81954 — Math — Linear Algebra

Question

Solve the following
0.6x + 0.8y + 0.1z = 1
1.1 x + 0.4y + 0.3z= 0.2
x + y + 2z =0.5
by LU decomposition method and find the inverse of the coefficient matrix

Solution

A = $\begin{pmatrix} 0.6 & 0.8 & 0.1 \\ 1.1 & 0.4 & 0.3 \\ 1 & 1 & 2 \end{pmatrix}$; b = $\begin{pmatrix} 1 \\ 0.2 \\ 0.5 \end{pmatrix}$

Swap some rows in matrices.

A = $\begin{pmatrix} 1 & 1 & 2 \\ 1.1 & 0.4 & 0.3 \\ 0.6 & 0.8 & 0.1 \end{pmatrix}$; b = $\begin{pmatrix} 0.5 \\ 0.2 \\ 1 \end{pmatrix}$

LU = A

L = $\begin{pmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{pmatrix}$; U = $\begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix}$;

$$\begin{pmatrix} 1 & 1 & 2 \\ 1.1 & 0.4 & 0.3 \\ 0.6 & 0.8 & 0.1 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.2 \\ 1 \end{pmatrix} \to R_2^* = R_2 + (-1.1) * R_1;\ R_3^* = R_3 + (-0.6) * R_1; \to$$
$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & -0.7 & -1.9 \\ 0 & 0.2 & -1.1 \end{pmatrix}\begin{pmatrix} 0.5 \\ -0.35 \\ 0.7 \end{pmatrix};$$

U = $\begin{pmatrix} 1 & 1 & 2 \\ 0 & -0.7 & -1.9 \\ 0 & 0 & -1.643 \end{pmatrix}$; b = $\begin{pmatrix} 0.5 \\ -0.35 \\ 0.6 \end{pmatrix}$

L = $\begin{pmatrix} 1 & 0 & 0 \\ 1.1 & 1 & 0 \\ 0.6 & * & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & -0.7 & -1.9 \\ 0 & 0.2 & -1.1 \end{pmatrix}\begin{pmatrix} 0.5 \\ -0.35 \\ 0.7 \end{pmatrix} \to R_3^* = R_3 + \left(\frac{0.2}{0.7}\right) * R_2 \to \begin{pmatrix} 1 & 1 & 2 \\ 0 & -0.7 & -1.9 \\ 0 & 0 & -1.643 \end{pmatrix}\begin{pmatrix} 0.5 \\ -0.35 \\ 0.6 \end{pmatrix}$$

L = $\begin{pmatrix} 1 & 0 & 0 \\ 1.1 & 1 & 0 \\ 0.6 & -0.2857 & 1 \end{pmatrix}$

U = $\begin{pmatrix} 1 & 1 & 2 \\ 0 & -0.7 & -1.9 \\ 0 & 0 & -1.643 \end{pmatrix}$; b = $\begin{pmatrix} 0.5 \\ -0.35 \\ 0.6 \end{pmatrix}$

$$\begin{cases} x + y + 2z = 0.5 \\ -0.7y - 1.9z = -0.35 \\ -1.643z = 0.6 \end{cases}$$

So, we have: Z =0.6/(-1.643) = -0.365

Y = $\frac{-0.35+1.9*(-0.365)}{-0.7}$= 1.4907

X = $0.5 - 2 * (-0.365) - 1.4907 = -0.2607$

Find the inverse of the coefficient matrix

$$\left(\begin{array}{ccc|ccc} 0.6 & 0.8 & 0.1 & 1 & 0 & 0 \\ 1.1 & 0.4 & 0.3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right);$$

1. $R_1^* = R_1/0.6$

$$\left(\begin{array}{ccc|ccc} 1 & 1.33 & 0.167 & 1.67 & 0 & 0 \\ 1.1 & 0.4 & 0.3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right);$$

2. $R_2^* = R_2 - 1.1 * R_1; R_3^* = R_3 - R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 1.33 & 0.167 & 1.67 & 0 & 0 \\ 0 & -1.067 & 0.1167 & -1.83 & 1 & 0 \\ 0 & -0.33 & 1.83 & -1.67 & 0 & 1 \end{array}\right);$$

3. $R_2^* = R_2/(-1.067)$

$$\left(\begin{array}{ccc|ccc} 1 & 1.33 & 0.167 & 1.67 & 0 & 0 \\ 0 & 1 & -0.109 & 1.719 & -0.9375 & 0 \\ 0 & -0.33 & 1.83 & -1.67 & 0 & 1 \end{array}\right);$$

4. $R_1^* = R_1 - 1.33 * R_2; R_3^* = R_3 + 0.33 * R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0.3125 & -0.625 & 1.25 & 0 \\ 0 & 1 & -0.109 & 1.719 & -0.9375 & 0 \\ 0 & 0 & 1.797 & -1.094 & -0.3125 & 1 \end{array}\right);$$

5. $R_3^* = \frac{R_3}{1.797}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0.3125 & -0.625 & 1.25 & 0 \\ 0 & 1 & -0.109 & 1.719 & -0.9375 & 0 \\ 0 & 0 & 1 & -0.609 & -0.174 & 0.556 \end{array}\right);$$

6. $R_1^* = R_1 - 0.3125 * R_3; R_2^* = R_2 + 0.109 * R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.435 & 1.304 & -0.174 \\ 0 & 1 & 0 & 1.65 & -0.957 & 0.061 \\ 0 & 0 & 1 & -0.609 & -0.174 & 0.556 \end{array}\right);$$

Answer: Z = -0.365; Y = 1.4907; X = -0.2607; $\begin{pmatrix} -0.435 & 1.304 & -0.174 \\ 1.65 & -0.957 & 0.061 \\ -0.609 & -0.174 & 0.556 \end{pmatrix}$.