Answer on Question #81889 – Math – Statistics and Probability

Question

Two players A and B toss a coin alternately. A begins the game and the player who first throws heads is the winner. B's coin is fair but A's is biased and has probability 'p' of showing heads. Find the value of 'p' so that the game is equiprobable to both players.

Solution

Let p be the probability of showing heads for A's coin.

Player A can win:

on his first throw – with probability p ;

on his second throw – with probability $(1-p)\cdot\frac{1}{2}\cdot p=\left(\frac{1-p}{2}\right)^{1}\cdot p$ (A throws tails – probability $1-p$, B throws tails – probability $\frac{1}{2}$, A throws heads);

on his third throw – with probability $(1-p)\cdot\frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}\cdot p=\left(\frac{1-p}{2}\right)^{2}\cdot p$ (A throws tails, B throws tails, A throws tails, B throws tails, A throws heads);

...

on his n-th throw – with probability $(1-p)\cdot\frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}\cdot\ldots\cdot(1-p)\cdot\frac{1}{2}\cdot p=\left(\frac{1-p}{2}\right)^{n-1}\cdot p$ (A throws tails, B throws tails, ..., A throws tails, B throws tails, A throws heads).

Then the probability to win for A is

$$\sum_{n=1}^{\infty} p\left(\frac{1-p}{2}\right)^{n-1}=p\cdot\frac{1}{1-\frac{1-p}{2}}=\frac{2p}{1+p}$$

(we applied the formula for the sum of geometric series).

Since the game is equiprobable to both players, this probability should equal $\frac{1}{2}$:

$$\frac{2p}{1+p}=\frac{1}{2}$$

from which it follows that

$$p=\frac{1}{3}$$

Answer: $p=\frac{1}{3}$.