Question

Two players A and B toss a coin alternately. A begins the game and the player who first throws heads is the winner. B's coin is fair, but A's is biased and has probability p of showing heads. The value of p so that the game is equiprobable to both players.

Solution

Let A get the head in Nth trial to win the game.

Since he is flipping the coin in odd trials,

P(N=1) = p,

P(N=3) = $(1 - p) * 0.5 * p,$

P(N=5) = $(1 - p)^2 * 0.5^2 * p,$ and so on.

Thus, P(A wins) = $p + (1 - p) * 0.5 * p + (1 - p)^2 * 0.5^2 * p + \cdots$
= $\sum(0.5^{i-1} * (1 - p)^{i-1} * p)$ = $\frac{p}{1-(0.5*(1-p))}$ = $\frac{p}{0.5+0.5p}$

P(wins) = 0.5

$\frac{p}{0.5+0.5p}$ = 0.5

p = 0.25+0.25*p

0.75*p=0.25

p=1/3≈ 0.33.

Answer: p=1/3.