Answer on Question #81791 – Math – Statistics and Probability

Question

According to the historical data gathered by an insurance company, the probability of a fire is 0.01. A manufacturer of fire alarms has determined through testing that a certain fire alarm has a 0.1 conditional probability to go off given that there is no fire, and a 0.2 conditional probability to remain silent given that there is a fire. The meaning of "go off" is that a fire alarm makes a loud sound or a bomb explodes.

Event A is that a fire breaks out. Event B is that the fire alarm goes off.

(a) If you hear this fire alarm go off, what is the conditional probability that there is a fire? In other words, what is $P(A|B)$?

(b) What is the conjunction probability, $P(B^C \cap A^C)$?

Solution

(a) $P(A) = 0.01$

$P(B|A^C) = 0.1, P(B^C|A) = 0.2$

According to Bayes' rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Apply the total probability theorem

$$P(B) = P(B|A)P(A) + P(B|A^C)P(A^C)$$

$$P(B) = (1 - 0.2)(0.01) + 0.1(1 - 0.01) = 0.107$$

Hence,

$$P(A|B) = \frac{(1 - 0.2)(0.01)}{0.107} = \frac{8}{107}$$

(b) Conditional probability

$$P(B^C|A^C) = \frac{P(B^C \cap A^C)}{P(A^C)}$$

$$P(B^C \cap A^C) = P(B^C|A^C)P(A^C)$$

$$P(B^C \cap A^C) = (1 - 0.1)(1 - 0.01) = 0.891$$

Answer: a) $P(A|B) = \frac{8}{107}$; **b)** $P(B^C \cap A^C) = 0.891$.