Answer on Question #81713 – Math – Statistics and Probability

Three turns of the same appearance are given as follows urn A contains 5 red and 6 white balls, urn B contains 6 red and 4 white balls, urn C contains 3 red and 5 white balls. An urn is selected at random and ball is drawn from the urn.

Question

(A) What is probability that ball drawn is red, white?

Solution

Let E_A, E_B, E_C and R be the events defined as follows:
$E_A =$ urn A is chosen, $E_B =$ urn B is chosen, $E_C =$ urn C is chosen, and
$R =$ ball drawn is red.
Since there are three urns of the same appearance and one of three urns is chosen at random, therefore

$$P(E_A) = P(E_B) = P(E_C) = \frac{1}{3}$$

The probability of drawing a red ball from urn A is

$$P(R|E_A) = \frac{5}{5+6} = \frac{5}{11}$$

The probability of drawing a red ball from urn B is

$$P(R|E_B) = \frac{6}{6+4} = \frac{3}{5}$$

The probability of drawing a red ball from urn C is

$$P(R|E_C) = \frac{3}{3+5} = \frac{3}{8}$$

We are required to find $P(R)$.
By Law of total probability we have

$$P(R) = P(E_A)P(R|E_A) + P(E_B)P(R|E_B) + P(E_C)P(R|E_C)$$

$$P(R) = \frac{1}{3}\left(\frac{5}{11}\right) + \frac{1}{3}\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{3}{8}\right) = \frac{200 + 264 + 165}{1320} = \frac{629}{1320} \approx 0.4765$$

Le t W be the event that ball drawn is white.
If we are required to find $P(W)$, then

$$P(W|E_A) = \frac{6}{5+6} = \frac{6}{11}$$

$$P(W|E_B) = \frac{4}{6+4} = \frac{2}{5}$$

$$P(W|E_C) = \frac{5}{3+5} = \frac{5}{8}$$

By Law of total probability we have

$$P(W) = P(E_A)P(W|E_A) + P(E_B)P(W|E_B) + P(E_C)P(W|E_C)$$

$$P(W)=\frac{1}{3}\left(\frac{6}{11}\right)+\frac{1}{3}\left(\frac{2}{5}\right)+\frac{1}{3}\left(\frac{5}{8}\right)=\frac{240+176+275}{1320}=\frac{691}{1320}\approx 0.5235$$

Check

$$P(R)+P(W)=\frac{629}{1320}+\frac{691}{1320}=1$$

Question

(B) What is probability that ball is from urn A given that ball is red?

Solution

Let E_A, E_B, E_C and R be the events defined as follows:
E_A = urn A is chosen, E_B = urn B is chosen, E_C = urn C is chosen, and
R = ball drawn is red.
Since there are three urns of the same appearance and one of three urns is chosen at random, therefore

$$P(E_A)=P(E_B)=P(E_C)=\frac{1}{3}$$

The probability of drawing a red ball from urn A is

$$P(R|E_A)=\frac{5}{5+6}=\frac{5}{11}$$

The probability of drawing a red ball from urn B is

$$P(R|E_B)=\frac{6}{6+4}=\frac{3}{5}$$

The probability of drawing a red ball from urn C is

$$P(R|E_C)=\frac{3}{3+5}=\frac{3}{8}$$

We are required to find $P(E_A|R)$.
By Bayes' theorem we have

$$P(E_A|R)=\frac{P(E_A)P(R|E_A)}{P(E_A)P(R|E_A)+P(E_B)P(R|E_B)+P(E_C)P(R|E_C)}$$

$$P(E_A|R)=\frac{\frac{1}{3}\left(\frac{5}{11}\right)}{\frac{1}{3}\left(\frac{5}{11}\right)+\frac{1}{3}\left(\frac{3}{5}\right)+\frac{1}{3}\left(\frac{3}{8}\right)}=\frac{200}{200+264+165}=\frac{200}{629}\approx 0.3180$$

The probability that ball is from urn A given that ball is red

$$P(E_A|R)=\frac{200}{629}\approx 0.3180$$