Properties of Logarithms:

1. $\log_a 1 = 0$

You can verify why this works by changing to an exponential form and getting $a^0 = 1$ and anything to the zero power is 1. This property says that no matter what the base is, if you are taking the logarithm of 1, then the answer will always be 0.

2. $\log_a a = 1$

You can verify this by changing to an exponential form and getting $a^1 = a$. This property says that if the base and the number you are taking the logarithm of are the same, then your answer will always be 1.

3. $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$

This property allows you to take a logarithmic expression of two things that are multiplied, then you can separate those into two distinct expressions that are added together. You can also go the other way. Two log expressions that are added can be combined into a single log expression using multiplication.

4. $\log_a(u^n) = n\log_a u$

This property allows you to take a logarithmic expression involving two things that are divided, then you can separate those into two distinct expressions that are subtracted. You can also go the other way. Two log expressions that are subtracted can be combined into a single log expression using division.

5. $\log_a(uv) = \log_a u + \log_a v$

This property will be very useful in solving equations and application problems. It allows you to take the exponent in a logarithmic expression and bring it to the front as a coefficient. You can also go the other way and move a coefficient up so that it becomes an exponent.

So if properties 3, 4 and 5 can be used both ways, how do you know what should be done? That depends on the type of problem that is being asked. If you are being asked to combine log expressions into a single expression, you'll want to use the property from right to left. But if you are trying to break up a single log expression into its separate parts, you'll want to use the property from left to right.

i. Use the properties of logs to write $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$ as a single logarithmic expression.

Since this problem is asking us to combine log expressions into a single expression, we will be using the properties from right to left.

We usually begin these types of problems by taking any coefficients and writing them as exponents.

$$\frac{1}{2}\log_{10} x + 3\log_{10}(x+1) = \log_{10} x^{1/2} + \log_{10}(x+1)^3$$

Now there are two log terms that are added. We can combine those into a single log expression by multiplying the two parts together.

$$\log_{10} x^{1/2} + \log_{10}(x+1)^3 = \log_{10}\left(x^{1/2}(x+1)^3\right)$$

We have now condensed the original problem into a single logarithmic expression.

ii. Expand the expression $\log_{10}(5x^3y)$.

Since we are trying to break the original expression up into separate pieces, we will be using our properties from left to right.

We begin by taking the three things that are multiplied together and separating those into individual logarithms that are added together.

$$\log_{10}(5x^3y) = \log_{10}(5) + \log_{10}(x^3) + \log_{10}(y)$$

There is an exponent in the middle term which can be brought down as a coefficient. This gives us

$$\log_{10}(5) + \log_{10}(x^3) + \log_{10}(y) = \log_{10}(5) + 3\log_{10}(x) + \log_{10}(y)$$

There are no terms multiplied or divided nor are there any exponents in any of the terms. We have expanded this expressionas much as possible.