Answer on Question #81522 – Math – Combinatorics | Number Theory

Question

If P_n=6^n+8^n,
(P_83÷49) what is the remainder?

Solution

First use Euler theorem (6 and 8 are both mutually prime with 49):

$6^{\phi(49)} \equiv 1(\mathrm{mod}49)$,

$8^{\phi(49)} \equiv 1(\mathrm{mod}49)$.

Find

$\phi(49) = \phi(7^2) = 7^2 - 7 = 42.$

Then

$6^{42} \equiv 1(\mathrm{mod}49)$, $8^{42} \equiv 1(\mathrm{mod}49)$

from which

$6^{84} = (6^{42})^2 \equiv 1(\mathrm{mod}49)$, $8^{84} = (8^{42})^2 \equiv 1(\mathrm{mod}49).$

Then $6^{83} \equiv 6^{-1}(\mathrm{mod}49)$, $8^{83} \equiv 8^{-1}(\mathrm{mod}49)$

Next, from an equality

$49 = 6 \cdot 8 + 1$

we have

$6^{-1} \equiv -8(\mathrm{mod}49)$, $8^{-1} \equiv -6(\mathrm{mod}\ 49)$,

and then

$6^{83} + 8^{83} \equiv -14(\mathrm{mod}49) \equiv 35(\mathrm{mod}49).$

Answer: 35.

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