

Answer on Question #81502 – Math – Trigonometry

Question

Find exact value of the expression?

$\tan a = 24/7$, a lies in quadrant 3 and $\cos B = -15/17$, B lies in quadrant 2. Find $\sin(a+B)$

Solution

$$\sin(a+B) = \sin(a) \cos(B) + \cos(a) \sin(B),$$

$$\tan^2 a = \sin^2(a) / \cos^2(a) = (1 - \cos^2(a)) / \cos^2(a) = 1 / \cos^2(a) - 1,$$

$$1 / \cos^2(a) = 1 + \tan^2 a,$$

$$\cos^2(a) = 1 / (1 + \tan^2 a) = 1 / (1 + 24^2/7^2) = 1 / (625/49) = 49/625;$$

a lies in quadrant 3, hence $\cos(a) < 0$, $\sin(a) < 0$,

$$\cos(a) = -\sqrt{\frac{49}{625}} = -7/25, \quad \sin^2(a) = 1 - \cos^2(a) = 576/625, \quad \sin(a) = -\sqrt{\frac{576}{625}} = -24/25,$$

$$\sin^2(B) = 1 - \cos^2(B) = 1 - 225/289 = 64/289,$$

B lies in quadrant 2, hence $\sin(B) > 0$,

$$\sin(B) = \sqrt{\frac{64}{289}} = 8/17,$$

$$\begin{aligned} \sin(a+B) &= \sin(a) \cos(B) + \cos(a) \sin(B) = (-24/25) \cdot (-15/17) + (-7/25) \cdot (8/17) \\ &= (360 - 56) / 425 = 304/425 \end{aligned}$$

Answer: 304/425.