

Question #8148 $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 5x = te^{2t} \cos 3t$

Solution. The general solution of linear non homogeneous equation is sum of general solution of the respective homogeneous equation and any solution of non-homogeneous. The general solution of homogeneous is $x(t) = C_1e^{5t} + C_2e^{-t}$. The solution of non-homogeneous should be found in the form $e^{2t}(ax + b) \cos 3t + e^{2t}(cx + d) \sin 3t$. It can be verified by substituting, that $x_0(t) = -1/54e^{2t}(3t \cos(3t) - \sin 3t)$. Thus, the general solution is $x(t) = C_1e^{5t} + C_2e^{-t} - 1/54e^{2t}(3t \cos(3t) - \sin 3t)$.

Answer. $x(t) = C_1e^{5t} + C_2e^{-t} - 1/54e^{2t}(3t \cos(3t) - \sin 3t)$