Answer on Question #81460 – Math – Differential Equations

Question

1. Solve the following ODE using the power series method:

$$(x+2)y'' + xy' - y = 0$$

Solution

Let

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots$$

Then

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \ldots$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + \ldots$$

$$xy' = a_1 x + 2a_2 x^2 + 3a_3 x^3 + 4a_4 x^4 + \ldots$$

$$(x+2)y'' = 4a_2 + (2a_2 + 12a_3)x + (6a_3 + 24a_4)x^2 + \ldots$$

And finally:

$$(x+2)y'' + xy' - y = (4a_2 - a_0) + (2a_2 + 12a_3)x + (a_2 + 6a_3 + 24a_4)x^2 + \cdots$$

But since

$$(x+2)y'' + xy' - y = 0$$

we come to the set of equations:

$$\begin{cases} (4a_2 - a_0) = 0 \\ (2a_2 + 12a_3) = 0 \\ (a_2 + 6a_3 + 24a_4) = 0 \\ \ldots \end{cases}$$

Solving it we obtain a_i:

$$\begin{cases} a_2 = a_0/4 \\ a_3 = -a_0/24 \\ a_4 = 0 \\ \ldots \end{cases}$$

So, a_i $(i \neq 1)$ depends on a_0 only, and if we define

$$h(x) = 1 + \frac{x^2}{4} - \frac{x^3}{24} + \ldots$$

then all solutions could be found in the form of

$$y(x) = a_0 h(x) + a_1 x$$

Answer

$$y(x) = a_0 h(x) + a_1 x,$$

where

$$h(x) = 1 + \frac{x^2}{4} - \frac{x^3}{24} + \ldots$$