Relating exponential and logarithmic functions

There is a close relationship between exponential functions and logarithmic functions: each is the inverse of the other.

This means that if $y=a^x$, then $x=\log_a(y)$ (which is log to base a of y.

Recall the definition of $\log_a(b)$:

$\log_a(b)$ is the power you have to raise a (the base) to, in order to get b.

But that means that if $\log_a(b)=c$, then c is the power you have to raise a to, in order to get b. Put more simply, $a^c=b$.

So saying that $\log_a(b)=c$ is just another way of saying $a^c=b$.

Here's an example with numbers to make it clearer.

Let's go back to the old function $f(x)=3^x$. For x=2, we get $f(2)=3^2=9$. We can twist this equation around, and rewrite by saying "the power we have to raise 3 to, to get 9 is 2", or in other words,

$\log_3(9)=2$.

Why should we want to rewrite the relationship like this?

Well, as always it's because we want to get any unknown that we're trying to find **on its own**.

If we knew $f(x) = 3^x$ and we wanted to know what value of x would give us f(x)=12, then we have to find the value of x from the equation $3^x=12$. How can we find x from this? By rewriting the equation in terms of logs.

We know that x is the power we have to raise 3 to, to get 12, so

$x=\log_3(12)$.

If we want to calculate the value of x with a calculator, we need to rewrite $3^x=12$ in terms of logs to base 10, or logs to base e, as those are the bases most calculators work with.

To do this, we take logs to base 10 of both sides, to get:

$\log_{10}(3^x)=\log_{10}(12)$.

This simplifies to give

$x\log_{10}(3)=\log_{10}(12)$.

So $x=\log_{10}(12)/\log_{10}(3)$, which a calculator will work out.

In particular, the function ln(x) (which is another way of writing $\log_e(x)$) is the inverse function of exp(x) (which is another way of writing e^x).

So if we know some quantity, q(t), grows in time like e^t, then we can write their relationship as either:

$q(t)=e^t$, or

$t=\log_e(q)$.

Which way we choose to write their relationship depends on what we're trying to find and what we know.