

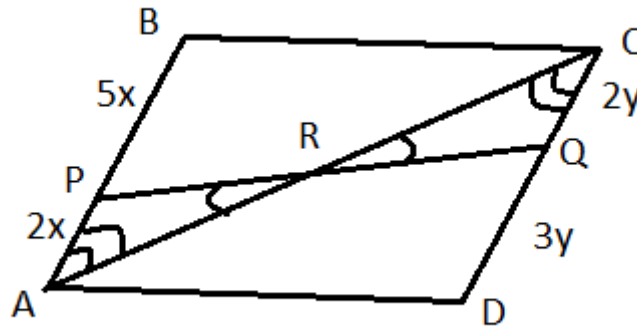
ANSWER on Question #81273 – Math – Geometry

QUESTION

In a parallelogram $ABCD$, P divides AB in the ratio $2 : 5$ and Q divides DC in the ratio $3 : 2$. If AC and PQ intersect at R . Find the ratio $AR : RC$ and $PR : RQ$.

SOLUTION

Draw pattern to this problem



Since P divides AB in the ratio $2 : 5$, then we introduce the proportionality coefficient x .

$$AP = 2x \text{ and } BP = 5x$$

Since Q divides DC in the ratio $2 : 3$, then we introduce the proportionality coefficient y .

$$CQ = 2y \text{ and } DQ = 3y$$

Since $ABCD$ is a parallelogram, then

$$AB = CD \rightarrow AP + PB = CQ + DQ \rightarrow 2x + 5x = 2y + 3y \rightarrow 7x = 5y \rightarrow \boxed{\frac{x}{y} = \frac{5}{7}}$$

Consider triangles $\triangle APR$ and $\triangle CQR$:

$$\angle PRA = \angle CRQ \text{ as a pair of vertical angles}$$

(More information: https://en.wikipedia.org/wiki/Angle#Vertical_and_adjacent_angle_pairs)

$$\angle PAR = \angle QCR \text{ as a pair of internal multi – faceted angles with } AB \parallel CD \text{ and } AC \text{ – secant}$$

Then,

$\Delta APR \sim \Delta CQR$ triangles are similar (AAA, angle angle angle)

$$\Delta APR \sim \Delta CQR \rightarrow \frac{AP}{CQ} = \frac{PR}{QR} = \frac{AR}{CR} \rightarrow \frac{2x}{2y} = \frac{PR}{QR} = \frac{AR}{CR} \rightarrow \frac{PR}{QR} = \frac{AR}{CR} = \frac{x}{y} = \frac{5}{7}$$

Conclusion,

$$\frac{PR}{QR} = \frac{5}{7} \rightarrow \boxed{PR : QR = 5 : 7}$$

$$\frac{AR}{CR} = \frac{5}{7} \rightarrow \boxed{AR : CR = 5 : 7}$$

ANSWER

$$PR : QR = 5 : 7$$

$$AR : CR = 5 : 7$$