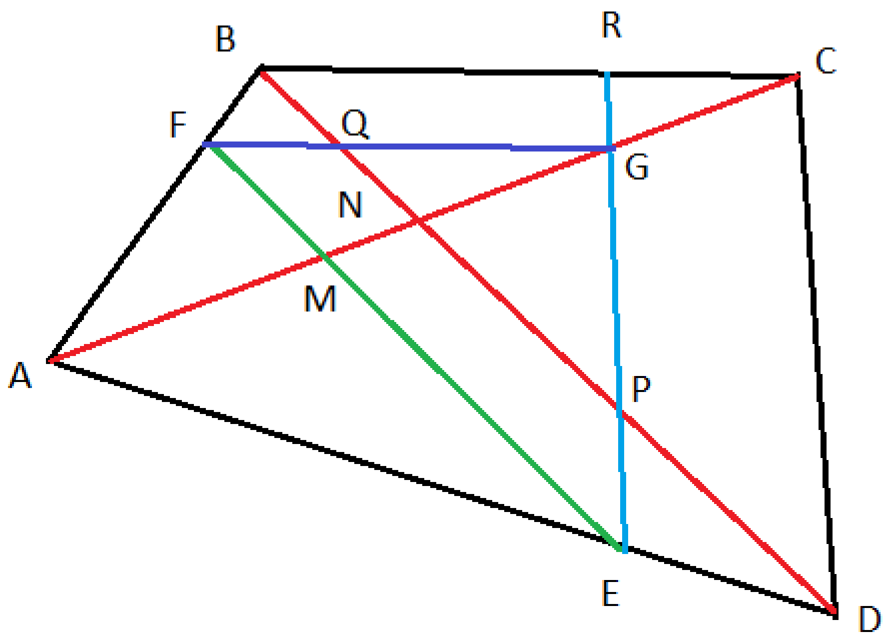
ANSWER on Question #81271 – Math – Geometry

QUESTION

$ABCD$ is a plane quadrilateral and E is any point on AD. EF is drawn parallel to DB to meet AB in F and EG is drawn parallel to DC to meet AC in G. Prove that FG is parallel to BC.

SOLUTION



$$EF \parallel BD \rightarrow \Delta AFE \sim \Delta ABD \rightarrow \frac{AF}{AB} = \frac{FE}{BD} = \frac{AE}{AD} = k$$

$$ER \parallel CD \rightarrow \Delta AGE \sim \Delta ACD \rightarrow \frac{AG}{AC} = \frac{GE}{CD} = \frac{AE}{AD} = m$$

Then,

$$k = \frac{AE}{AD} = m \rightarrow \boxed{m = k}$$

We have vectors $\overrightarrow{EF}$, $\overrightarrow{DB}$, $\overrightarrow{EG}$, $\overrightarrow{DC}$, $\overrightarrow{FG}$, and $\overrightarrow{BC}$.

$$\overrightarrow{FG} = \overrightarrow{EG} - \overrightarrow{EF}$$

$$\overrightarrow{BC} = \overrightarrow{DC} - \overrightarrow{DB}$$

$$\overrightarrow{DB} = \frac{1}{k} \cdot \overrightarrow{EF}$$

$$\overrightarrow{DC} = \frac{1}{m} \cdot \overrightarrow{EG} = \frac{1}{k} \cdot \overrightarrow{EG}$$

Then,

$$\overrightarrow{BC} = \overrightarrow{DC} - \overrightarrow{DB} = \frac{1}{k} \cdot \overrightarrow{EG} - \frac{1}{k} \cdot \overrightarrow{EF} = \frac{1}{k} \cdot \left(\overrightarrow{EG} - \overrightarrow{EF}\right) = \frac{1}{k} \cdot \overrightarrow{FG}$$

$$\boxed{\overrightarrow{BC} = \frac{1}{k} \cdot \overrightarrow{FG}}$$

Conclusion,

$$The\ vectors\ \overrightarrow{BC}\ and\ \overrightarrow{FG}\ are\ collinear\ vectors. Hence, BC \parallel FG$$

Q.E.D.