Answer on Question #81216 – Math – Calculus

Question

1. Integral of sin squared 2x cos raised to 4 2x dx

2. Integral of cot raised to 5 x dx

Solution

1.

$$\int \sin^2(2x)\cos^4(2x)dx = \int\big(\sin(2x)\cos(2x)\big)^2\cos^2(2x)dx =$$
$$= \int\left(\frac{1}{2}\sin(4x)\right)^2 \cdot \frac{1+\cos(4x)}{2}dx = \frac{1}{8}\int \sin^2(4x)dx + \frac{1}{8}\int \sin^2(4x)\cos(4x)dx =$$
$$= \frac{1}{16}\int\big(1-\cos(8x)\big)dx + \frac{1}{32}\int \sin^2(4x)d\sin(4x) =$$
$$= \frac{1}{16}x - \frac{1}{128}\sin(8x) + \frac{1}{96}\sin^3(4x) + C$$

2.

$$\int \cot^5 x dx = \int \frac{\cos^5 x}{\sin^5 x}dx = \int \frac{\cos^5 x \sin x dx}{\sin^6 x} = \begin{bmatrix}\sin^2 x = t \\ 2\sin x\cos dx = dt \\ \cos^2 x = 1-t^2\end{bmatrix} = \frac{1}{2}\int \frac{(1-t^2)^2 dt}{t^3} =$$
$$= \frac{1}{2}\int(t^{-3} - 2t^{-1} + t)dt = \frac{1}{2}\left(-\frac{1}{4}t^{-4} - 2\ln|t| + \frac{1}{2}t^2\right) + C = -\frac{1}{8\sin^8 x} - 2\ln|\sin x| + \frac{1}{4}\sin^2 x + C$$

Answer: 1. $\int \sin^2(2x)\cos^4(2x)dx = \frac{1}{16}x - \frac{1}{128}\sin(8x) + \frac{1}{96}\sin^3(4x) + C$;

2. $\int \cot^5 x dx = \frac{1}{16}x - \frac{1}{128}\sin(8x) + \frac{1}{96}\sin^3(4x) + C$