Answer on Question #81212 – Math – Functional Analysis

Question

show that absolute value of. a linear functional has properties of sublinear functional.

Solution

Let f be a linear functional $f : X \to \mathbb{R}.$

Consider $\varphi(x) = \|f(x)\|$.

We have for $\lambda \geq 0$:

$$\varphi(\lambda x) = \|f(\lambda x)\| = \|\lambda f(x)\| = \lambda \|f(x)\| = \lambda \varphi(x).$$

This proves that φ is nonnegatively homogeneous.

Then, for $x, y \in X$:

$$\varphi(x + y) = \|f(x + y)\| = \|f(x) + f(y)\| \leq \|f(x)\| + \|f(y)\| = \varphi(x) + \varphi(y).$$

This proves that φ is subadditive.

These two properties prove that φ is sublinear functional.