

**Answer on Question #81204 – Math – Discrete Mathematics
Question**

A) Find the generating function of the recurrence

$$a_n = 4a_{n-1} - 4a_{n-2} + 1$$

with initial conditions $a_0 = 1, a_1 = 1$

Solution

Compute

$$a_2 = 4a_1 - 4a_0 + 1 = 4 - 4 + 1 = 1,$$

$$a_3 = 4a_2 - 4a_1 + 1 = 4 - 4 + 1 = 1$$

and so on.

The generating function is given by the formula

$$A = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \dots = 1 + x + x^2 + x^3 + x^4 + \dots$$

Then

$$-xA = -x - x^2 - x^3 - x^4 - x^5 - \dots$$

Adding the right-hand sides of A and $-xA$ one gets

$$(1 - x)A = 1$$

Answer:

$$A = \frac{1}{1 - x}$$

Question

B) Express $3x^4 + 2x^3 - 2x^2 + x$ in terms of x^4, x^3, x^2 and x

Answer: 3, 2, -2, 1.