Answer on Question #81191 – Math – Statistics and Probability

Question

If a random variable U has the t-distribution with n degrees of freedom, show that Z=U^2 has the F-distribution with 1 and n degrees of freedom.

Solution

Student's t-distribution with *n* degrees of freedom can be defined as the distribution of the random variable T with

$$T = \frac{Z}{\sqrt{\frac{W}{n}}}$$

where Z is a standard normal distribution,

W is a chi-squared distribution with n degrees of freedom,

Z and W are independent.

Thus,

$$T^2 = \frac{Z^2}{\left(\frac{W}{n}\right)}.$$

Since Z^2 has a chi–square distribution with 1 degree of freedom, and Z^2 and W are independent, T^2 has an F distribution with 1 numerator and n denominator degrees of freedom.