Answer on Question #81144 – Math – Linear Algebra

Question

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_3, x_2 - x_3, x_1)$. Is T invertible? If yes, find a rule for T^{-1} like the one which defines T.

Solution

If $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 - x_3, x_2 - x_3, x_1)$, we can define $T = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}.$ $Det(T) = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} - 0 - 1 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 + 0 - (0 - 1) = 1 \neq 0$ Then T is invertible.

Augment the matrix T with identity matrix

/1	Ŏ	-1	1	0	0\	
0	1	-1	0	1	0	
\backslash_1	0	0	0	0	1/	

$\begin{pmatrix} 1\\ 0\\ 1 \end{pmatrix}$	0 1 0	$-1 \\ -1 \\ 0$	1 0 0	0 1 0	$\begin{pmatrix} 0\\0\\1 \end{pmatrix} \xrightarrow{R_3 - R_1}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	0 · 1 · 0	-1 -1 1	1 0 -1	0 1 0	$\begin{pmatrix} 0\\0\\1 \end{pmatrix}$
$\begin{pmatrix} 1\\ 0\\ 0 \end{pmatrix}$	0 1 0	-1 -1 1	$ \begin{array}{c} 1 \\ 0 \\ -1 \end{array} $	0 1 0	$\begin{pmatrix} 0\\0\\1 \end{pmatrix} \frac{R_1 + R_2}{R_1 + R_2}$	$\stackrel{3}{\rightarrow} \begin{pmatrix} 1\\0\\0 \end{pmatrix}$	0 1 0	$0 \\ -1 \\ 1$	$0 \\ 0 \\ -1$	0 1 0	$\begin{pmatrix} 1\\0\\1 \end{pmatrix}$
$\begin{pmatrix} 1\\ 0\\ 0 \end{pmatrix}$	0 1 0	0 -1 1	0 0 -1	0 1 0	$\begin{pmatrix} 1\\0\\1 \end{pmatrix} \frac{R_2 + R_2}{m_1}$	$\stackrel{R_3}{\to} \begin{pmatrix} 1\\0\\0\\0 \end{pmatrix}$	0 1 0	0 0 1	0 -1 -1	0 1 0	$\begin{pmatrix} 1\\1\\1 \end{pmatrix}$

As can be seen, we have obtained the identity matrix to the left. So, we are done. $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$T^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

 T^{-1} : $\mathbb{R}^3 \to \mathbb{R}^3$ can be defined by

$$T^{-1}(x_1, x_2, x_3) = (x_3, -x_1 + x_2 + x_3, -x_1 + x_3).$$

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