Answer on Question #81142 – Math – Linear Algebra

Question

Complete the set S = {x
3 +x
2 +1, x
2 +x+1, x+1} to get a basis of P3

Solution

Denote

$a_1 = x^3 + x^2 + 1, a_2 = x^2 + x + 1, a_3 = x + 1.$

Let's check the hypothesis that a_1, a_2, a_3 and $a_4 = 1$ form a basis in P_3. We have to check whether $c_1a_1 + c_2a_2 + c_3a_3 + c_4a_4 = 0$

yields

$c_1 = c_2 = c_3 = c_4 = 0.$

We have

$c_1(x^3 + x^2 + 1) + c_2(x^2 + x + 1) + c_3(x + 1) + c_4 \cdot 1 = 0$

This means

$c_1x^3 + (c_1 + c_2)x^2 + (c_2 + c_3)x + (c_1 + c_2 + c_3 + c_4) = 0.$

Then

$$\begin{cases} c_1 = 0 \\ c_1 + c_2 = 0 \\ c_2 + c_3 = 0 \\ c_1 + c_2 + c_3 + c_4 = 0 \end{cases}$$

from which

$c_1 = 0, c_2 = -c_1 = 0, c_3 = -c_2 = 0, c_4 = -c_1 - c_2 - c_3 = 0.$

This means that a_1, a_2, a_3, a_4 really form a basis in P_3.

Answer: 1 should be added, the basis of P_3 is $\{x^3 + x^2 + 1, x^2 + x + 1, x + 1, 1\}$.