

Answer on Question #81122 – Math – Calculus

Question

$f(x) = x^3 + 3$, $a = 6$. Evaluate $f'(a)$.

Solution

By the definition of the derivative of the function at the point $x = a$ we have

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Here $a = 6$, so we get

$$f'(6) = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h}.$$

Note that

$$f(6) = 6^3 + 3,$$

$$f(6+h) = (6+h)^3 + 3,$$

$$f(6+h) - f(6) = (6+h)^3 + 3 - (6^3 + 3) = (6+h)^3 + 3 - 6^3 - 3 = (6+h)^3 - 6^3.$$

Using $u^3 - v^3 = (u-v)(u^2 + uv + v^2)$ we get

$$(6+h)^3 - 6^3 = (6+h-6)((6+h)^2 + 6(6+h) + 6^2) = h((6+h)^2 + 6(6+h) + 6^2), \text{ so}$$

$$f(6+h) - f(6) = h((6+h)^2 + 6(6+h) + 6^2) \text{ and}$$

$$f'(6) = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}((6+h)^2 + 6(6+h) + 6^2)}{\cancel{h}} = \lim_{h \rightarrow 0} ((6+h)^2 + 6(6+h) + 6^2) = \left[\begin{array}{l} (6+h)^2 \rightarrow 36 \text{ as } h \rightarrow 0 \\ 6(6+h) \rightarrow 36 \text{ as } h \rightarrow 0 \\ 6^2 \rightarrow 36 \text{ as } h \rightarrow 0 \end{array} \right] =$$

$$= 36 + 36 + 36 = 108.$$

Answer: 108.