

## Answer on Question #81122 – Math – Calculus

### Question

$f(x) = x^3 + 3$ ,  $a = 6$ . Evaluate  $f'(a)$ .

### Solution

By the definition of the derivative of the function at the point  $x = a$  we have

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Here  $a = 6$ , so we get

$$f'(6) = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h}.$$

Note that

$$f(6) = 6^3 + 3,$$

$$f(6+h) = (6+h)^3 + 3,$$

$$f(6+h) - f(6) = (6+h)^3 + 3 - (6^3 + 3) = (6+h)^3 + 3 - 6^3 - 3 = (6+h)^3 - 6^3.$$

Using  $u^3 - v^3 = (u-v)(u^2 + uv + v^2)$  we get

$$(6+h)^3 - 6^3 = (6+h-6)((6+h)^2 + 6(6+h) + 6^2) = h((6+h)^2 + 6(6+h) + 6^2), \text{ so}$$

$$f(6+h) - f(6) = h((6+h)^2 + 6(6+h) + 6^2) \text{ and}$$

$$\begin{aligned} f'(6) &= \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} = \\ &= \lim_{h \rightarrow 0} \frac{h((6+h)^2 + 6(6+h) + 6^2)}{h} = \lim_{h \rightarrow 0} ((6+h)^2 + 6(6+h) + 6^2) = \begin{cases} (6+h)^2 \rightarrow 36 \text{ as } h \rightarrow 0 \\ 6(6+h) \rightarrow 36 \text{ as } h \rightarrow 0 \\ 6^2 \rightarrow 36 \text{ as } h \rightarrow 0 \end{cases} = \\ &= 36 + 36 + 36 = 108. \end{aligned}$$

**Answer:** 108.