

Answer on Question #81078 – Math – Trigonometry

Question

$$\cos^{\pi/8} + \cos^{3\pi/8} + \cos^{5\pi/8} + \cos^{7\pi/8} = 2$$

Solution

$$\text{Since } \cos \alpha + \cos \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\begin{aligned} \cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right) \\ &= 2 \cdot \cos\left(\frac{\frac{7\pi}{8} + \frac{\pi}{8}}{2}\right) \cos\left(\frac{\frac{7\pi}{8} - \frac{\pi}{8}}{2}\right) + 2 \cdot \cos\left(\frac{\frac{5\pi}{8} + \frac{3\pi}{8}}{2}\right) \cos\left(\frac{\frac{5\pi}{8} - \frac{3\pi}{8}}{2}\right) \\ &= 2 \cdot \cos\left(\frac{4\pi}{8}\right) \cos\left(\frac{3\pi}{8}\right) + 2 \cdot \cos\left(\frac{4\pi}{8}\right) \cos\left(\frac{\pi}{8}\right) \\ &= 2 \cdot \cos\left(\frac{\pi}{2}\right) \cdot \left(\cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right)\right) = 0 \end{aligned}$$

$$\text{because } \cos\left(\frac{\pi}{2}\right) = 0$$

$$\text{Answer: } \cos(\pi/8) + \cos(3\pi/8) + \cos(5\pi/8) + \cos(7\pi/8) = 0$$