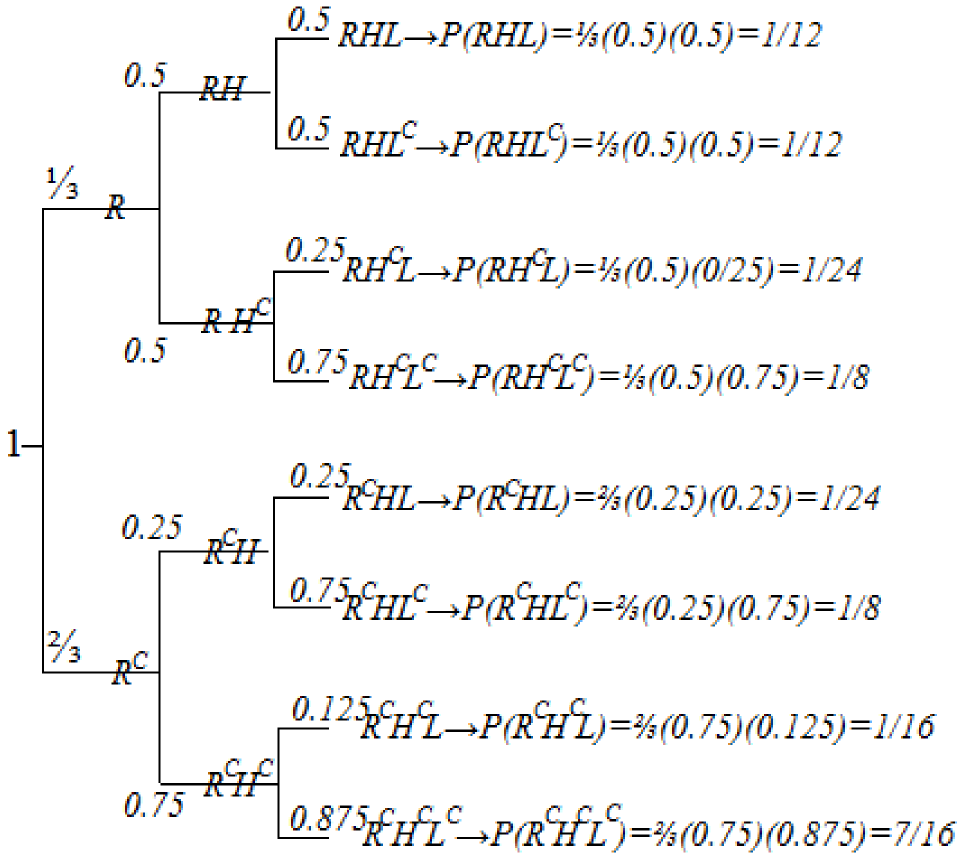
Answer on Question #81057 – Math – Statistics and Probability
Question

Liza lives in a small town. In her town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 0.5 and given that it is not rainy, there will be heavy traffic with probability 0.25. If it's rainy and there is heavy traffic, Liza arrives late for work with probability 0.5. On the other hand, the probability of being late is reduced to 0.125 if it is not rainy and there is no heavy traffic. In other situation (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. Assume Liza pick a random day.

i. What is the probability that it's not raining and there is heavy traffic and Liza is not late for work?
ii. What is the probability that Liza is late for work?
iii. Given that Liza arrived late at work, what is the probability that it rained that day?

Solution

Let R be the event that it is rainy, H be the event that there is heavy traffic, and L be the event that Liza is late for work.
Construct tree diagram



Each leaf in the tree corresponds to a single outcome in the sample space. We can calculate the probabilities of each outcome in the sample space by multiplying the probabilities on the edges of the tree that lead to the corresponding outcome.

i. The probability that it's not raining and there is heavy traffic and Liza is not late for work can be found applying the chain rule:

$$P(R^C \cap H \cap L^C) = P(R^C)P(H|R^C)P(L^C|R^C \cap H) = \frac{2}{3}(0.25)(0.75) = \frac{1}{8}$$

ii. The probability that Liza is late for work can be found using the law of total probability:

$$P(L) = P(R,H,L) + P(R,H^C,L) + P(R^C,H,L) + P(R^C,H^C,L) =$$

$$= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{11}{48}$$

iii. We can find the probability that it rained that day given that Liza arrived late at work $P(R|L)$ using

$$P(R|L) = \frac{P(R \cap L)}{P(L)}$$

$$P(R \cap L) = P(R,H,L) + P(R,H^C,L) = \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$$

$$P(L) = \frac{11}{48}$$

Then

$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{\frac{1}{8}}{\frac{11}{48}} = \frac{6}{11}.$$