Answer on Question #81004 – Math – Abstract Algebra

Question

Let G be a group and $\leq G$. Prove that the only right coset of H in G that is a subgroup of G is H itself.

Solution

By definition, $Hg = \{hg: h \in H\}$ is the right coset of H in G with respect to $g \in G$. Assume that Hg is a subgroup.
According to the definition of a group, $1 \in Hg$. Therefore, $g^{-1} = 1 * g^{-1} = h_1 g g^{-1} = h_1 \in H$.
According to the definition of a group, $g = (g^{-1})^{-1} \in H$, and for this reason $Hg = H$ (in this case, $hg \in H$ and $h = (hg^{-1})g \in Hg$, i. e. $Hg \subset H$ and $H \subset Hg$).

Answer

If Hg is a sugbroup, then $g \in H$ and $Hg = H$.