Answer on Question #81001 – Math – Abstract Algebra

Question

Check whether or not $\{z \in \mathbb{C} \mid z^5 = 1\}$ is a group with respect to addition.

Solution

From the parallelogram law (en.wikipedia.org/wiki/Parallelogram_law#Proof), if an angle between two vectors is less then $90°$, then the length of the sum of two vectors is greater than their lengths.
The solutions to this equation are 5 complex unit vectors $z_k = e^{2i\pi k/5}$ (the angle is divisible by 72), and therefore the addition group operation for two neighbour roots can't be defined. The set is not a group with respect to addition.

Answer:

It is not a group.