Answer on Question #80925 – Math – Linear Algebra

Question

Verify that W is a subspace of V. Assume that V has the standard operations.

$W = \{(x_1, x_2, x_3, 0)\}$, where x_1, x_2, x_3 are real numbers.
$V = \mathbb{R}^4$.

Solution

W is a subspace, if the following three conditions are satisfied:
1) W is non-empty (the zero vector is in W).
2) W is closed under addition: if $\vec{u}$ and $\vec{w}$ are in W, then $\vec{u} + \vec{w}$ is in W.
3) W is closed under scalar multiplication: if $\vec{u}$ is in W, and c is a scalar, then $c\vec{u} \in W$.

1) W is non-empty because it contains the zero vector $(0, 0, 0, 0)$.

2) Let $\vec{u} = (u_1, u_2, u_3, 0)$ and $\vec{w} = (w_1, w_2, w_3, 0)$ be two vectors in W. Show that W is closed under addition
$\vec{u} + \vec{w} = (u_1, u_2, u_3, 0) + (w_1, w_2, w_3, 0) = (u_1 + w_1, u_2 + w_2, u_3 + w_3, 0) =$
$= (x_1, x_2, x_3, 0)$
where $x_1 = u_1 + w_1, x_2 = u_2 + w_2$ and $x_3 = u_3 + w_3$ are real numbers.
Hence, $\vec{u} + \vec{w}$ is in W.

3) Let $\vec{u} = (u_1, u_2, u_3, 0)$ be a vector in W, and let c be any real number. Show that W is closed under scalar multiplication
$\overline{c\vec{u}} = c(u_1, u_2, u_3, 0) = (cu_1, cu_2, cu_3, 0) = (x_1, x_2, x_3, 0)$
where $x_1 = cu_1, x_2 = cu_2$ and $x_3 = cu_3$ are real numbers.
Hence, $c\vec{u}$ is in W.

Finally, because all three conditions are satisfied, we can conclude that W is a subspace of $\mathbb{R}^4$.

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