

Answer on Question #80900 – Math – Calculus

Question

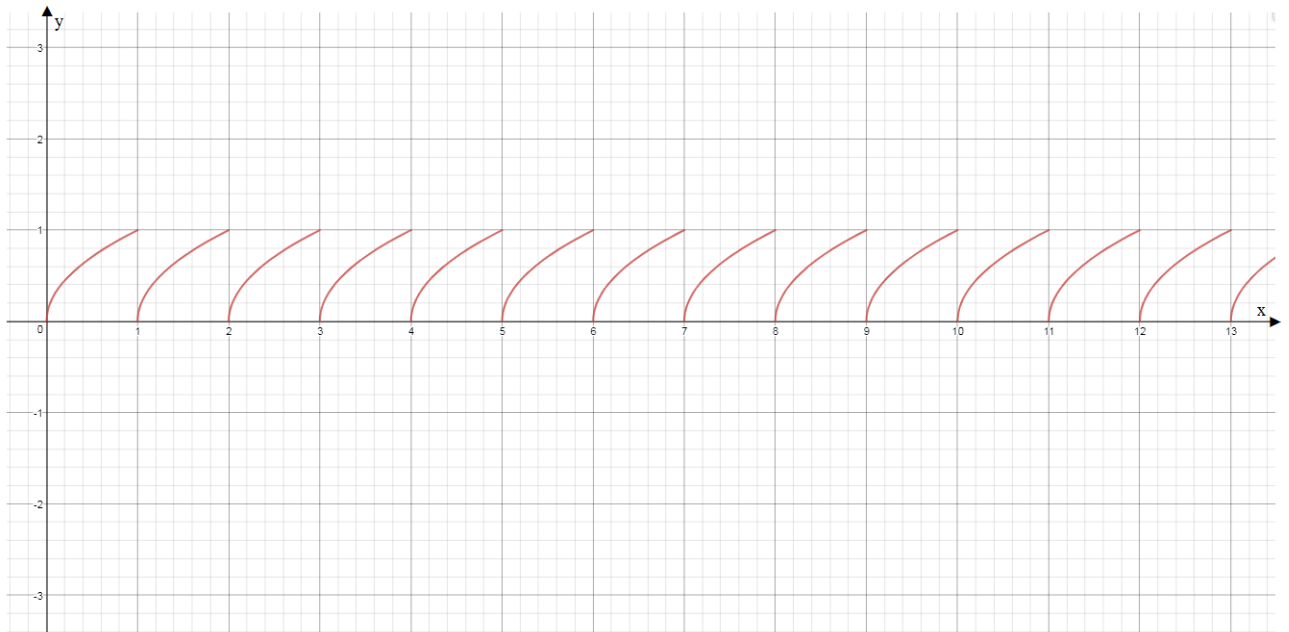
Definite integral

$$\int_0^{100} \sqrt{\{x\}} dx,$$

where $\{x\}$ is the fractional part of x .

Solution

If $x \geq 0$, $\{x\} = \text{frac}(x) = x - [x]$



We see that

$$\int_0^1 \sqrt{\{x\}} dx = \int_1^2 \sqrt{\{x\}} dx = \int_2^3 \sqrt{\{x\}} dx = \dots = \int_{98}^{99} \sqrt{\{x\}} dx = \int_{99}^{100} \sqrt{\{x\}} dx$$

Then

$$\begin{aligned} \int_0^{100} \sqrt{\{x\}} dx &= 100 \int_0^1 \sqrt{\{x\}} dx = 100 \lim_{A \rightarrow 1^-} \int_0^A \sqrt{\{x\}} dx = \\ &= 100 \lim_{A \rightarrow 1^-} \int_0^A \sqrt{x - [x]} dx = 100 \lim_{A \rightarrow 1^-} \int_0^A \sqrt{x - 0} dx = 100 \lim_{A \rightarrow 1^-} \left[\frac{2}{3} x^{3/2} \right]_0^A = \\ &= 100 \lim_{A \rightarrow 1^-} \left(\frac{2}{3} A^{3/2} - 0 \right) = 100 \left(\frac{2}{3} \right) = \frac{200}{3}. \end{aligned}$$

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