Answer on Question #80890 – Math – Linear Algebra

Question

Let P^(e)(x) = {p(x) ∈ R[x]|p(x) = p(−x)} Find W =P(e)∩P3. Find a basis for W and the dimension of W.

Solution

P_3 consists of all polynomials of degree 3, i.e. of all functions

$$y = ax^3 + bx^2 + cx + d$$

$y \in P(e)$ means

$$ax^3 + bx^2 + cx + d = a(-x)^3 + b(-x)^2 + c(-x) + d$$

from which

$$ax^3 + cx = 0$$

This holds for all x only if $a = c = 0$.

So

$$W = \{y = bx^2 + d, b \in \mathbb{R}, d \in \mathbb{R}\}.$$

A basis for W is $\{1, x^2\}$.

Indeed, these two elements are linearly independent since

$\alpha + \beta x^2 = 0$ for all x yields $\alpha = \beta = 0$

and all elements of W are linear combinations of 1 and x^2.

Hence the dimension of W is 2.

Answer: $W = \{y = bx^2 + d, b \in \mathbb{R}, d \in \mathbb{R}\}$; a basis for W is $\{1, x^2\}$; $dim(W) = 2$.