Answer on Question #80848 – Math – Linear Algebra

Question

Let T : $R^3 \to R^3$ be defined by T (x1, x2, x3) = (x1 −x3, x2 −x3, x1). Is T invertible? If yes, find a rule for T^{-1} like the one which defines T?

Solution

We need to decide if the solution to

$T(y_1, y_2, y_3) = (x_1, x_2, x_3)$

exists and is unique for all $(x_1, x_2, x_3) \in \mathbb{R}^3$.

This means

$$\begin{cases} y_1 - y_3 = x_1 \\ y_2 - y_3 = x_2 \\ y_1 = x_3 \end{cases}$$

Then

$$\begin{cases} y_1 = x_3 \\ y_3 = x_3 - x_1 \\ y_2 = x_2 + y_3 = x_2 + x_3 - x_1 \end{cases}$$

The unique solution exists for any x_1, x_2, x_3. The rule is

$T^{-1}(x_1, x_2, x_3) = (x_3, x_2 + x_3 - x_1, x_3 - x_1)$.

Answer: Yes, it is invertible. $T^{-1}(x_1, x_2, x_3) = (x_3, x_2 + x_3 - x_1, x_3 - x_1)$.