Answer on Question #80774 – Math – Calculus

Question

$$\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx$$

Solution

We are given

$$\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx$$

We know that

$$1+\cos x = 2\cos^2\frac{x}{2}$$
$$1-\cos x = 2\sin^2\frac{x}{2}$$

Then we have

$$\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx = \int \frac{\sqrt{2\cos^2\frac{x}{2}}}{\left(2\sin^2\frac{x}{2}\right)^{5/2}}dx = \int \frac{\sqrt{2}\cos\frac{x}{2}}{(2)^{5/2}\sin^5\frac{x}{2}}dx = \int \frac{\cos\frac{x}{2}}{4\sin^5\frac{x}{2}}d(x) =$$

$$\frac{1}{2}\int \frac{\cos\frac{x}{2}}{\sin^5\frac{x}{2}}d\left(\frac{x}{2}\right) = \frac{1}{2}\int \frac{d\left(\sin\frac{x}{2}\right)}{\sin^5\frac{x}{2}} = \left|t = \sin\frac{x}{2}\right| = \frac{1}{2}\int \frac{dt}{t^5} = \frac{1}{2}\int t^{-5}dt =$$

Apply power rule: $\int u^n dt = \frac{u^{n+1}}{n+1} + C$

$$= -\frac{1}{2}\frac{t^{-4}}{4} + C = -\frac{1}{8t^4} + C = -\frac{1}{8\sin^4\left(\frac{x}{2}\right)} + C$$

Answer: $\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx = -\frac{1}{8\sin^4\left(\frac{x}{2}\right)} + C$