Question 1. *Show that $f = O(1)$ as $x \to a$ if and only if $f(x)$ is bounded on some neighborhood of a.*

Solution. Recall that $f(x) = O(g(x))$ as $x \to a$ iff there are $M > 0$ and $\delta > 0$, such that $|f(x)| \leq M|g(x)|$ for all x with $|x - a| < \delta$. Use this definition in the case when $g(x) \equiv 1$ and obtain that there are $M > 0$ and $\delta > 0$ such that $|f(x)| \leq M$ for all x, such that $|x - a| < \delta$. This is precisely the statement that f is bounded by M in the δ-neighborhood of a. $\square$