Question 1. *Prove the statement "If $f = o(g)$ then $f = O(g)$".*

Solution. Recall that $f(x) = o(g(x))$ as $x \to x_0$ iff for any $\varepsilon > 0$ there is $\delta > 0$, such that $|f(x)| \leq \varepsilon|g(x)|$ for all x, such that $|x - x_0| < \delta$. Also recall that $f(x) = O(g(x))$ iff there are $M > 0$ and $\delta > 0$, such that $|f(x)| \leq M|g(x)|$ for all x with $|x - x_0| < \delta$. Now if $f(x) = o(g(x))$ as $x \to x_0$, then one can fix some $\varepsilon > 0$, find the corresponding $\delta > 0$ and set $M = \varepsilon$. Then $|f(x)| \leq M|g(x)|$ for all x, such that $|x - x_0| < \delta$. By definition, this means that $f(x) = O(g(x))$. $\square$