Question #8056

Solution. Consider circle C of center $I(1,1)$ and radius $R=1$ and the circle C' of center $I'(5,1)$ and $R'=2$. Write the equation of the common tangents to C and C' using scalar product.

Answer. I doubt that scalar product helps in this problem. However, you can do as the following. The tangent line $y=kx+l$ has only one point with the circle, hence the corresponding quadratic equation has only one root, which is equivalent to that determinant of respective equations is zero. The equation of the first circle is $(x-1)^2+(y-1)^2=1$, of the second — $(x-5)^2+(y-1)^2=4$. Put $y=kx+l$ into those equations. One can obtain: $x^2(1+k^2)-2x(1+k)+(l-1)^2=0$, $x^2(1+k^2)-2x(5+k)+21+(l-1)^2=0$ and we get two equations on k,l: $(1+k)^2=(l-1)^2(1+k^2)$ and $(5+k)^2=(1+k^2)(21+(l-1)^2)$. These two conditions determine all common tangents.