Answer on Question #80554 - Math - Abstract Algebra

Question. Not every polynomial that is irreducible over $\mathbb{Q}[x]$ is irreducible over $\mathbb{Z}[x]$. State whether the statement is true or false, justify with reason.

Answer. True. The statement is equivalent to saying that there is a polynomial that is irreducible in $\mathbb{Q}[x]$ and is not irreducible in $\mathbb{Z}[x]$. A polynomial $p = 3x + 6$ is an example of such a polynomial.

In $\mathbb{Z}[x]$, $p = 3(x + 2)$. As $\mathbb{Z}$ is an integral domain, the set of units in $\mathbb{Z}[x]$ is the same as in $\mathbb{Z}$, namely, $\{-1, 1\}$. Hence neither of the factors of p is a unit, and p is not irreducible.

Assume that p factorizes as q_1q_2 in $\mathbb{Q}[x]$. As $\mathbb{Q}$ is an integral domain, the set of units in $\mathbb{Q}[x]$ is the set of all non-zero elements of $\mathbb{Q}$. As p is not zero, both q_1 and q_2 are not zero. As $\mathbb{Q}$ is an integral domain, the sum of the degrees of q_1 and q_2 is the degree of p which is 1. Hence for some $i \in \{1, 2\}$, q_i has degree 0. Hence q_i is a unit in $\mathbb{Q}[x]$. Also p is not a unit. We conclude that p is irreducible in $\mathbb{Q}[x]$.

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