Answer on Question #80552 - Math - Abstract Algebra

Question. The function f, defined by

$$f(x) = \frac{x-1}{3-x},$$

has the same set as domain and as range. State whether the statement is true or false, justify the answer with reason.

Answer. False. The range of f does not contain -1 for the following reason. Assume that $f(x) = -1$ for some x. Then, as $f(x)$ has a value, $3 - x \neq 0$. Hence $\frac{x-1}{3-x} = -1$, $x - 1 = (-1)(3 - x) = x - 3$, $-1 = -3$, contradiction. Therefore, $f(x) \neq -1$ for all x.

The domain of f contains -1 because if $x = -1$, then $3 - x = 4 \neq 0$, so the division in the formula of f is defined, and f is defined at -1.

If the range and the domain of f were the same set, then the range of f would contain -1 because the domain of f contains it. But the range of f does not contain -1, contradiction. Therefore, the range and the domain of f are not the same set.