Question

Obtain eqñ of parabola with focus (3,2)
Directrix 3x-4y+9=0

Solution

The parabola is the locus of points in that plane that are equidistant from both the directrix and the focus.

The distance from point (x, y) to the point (3,2) is $\sqrt{(x-3)^2 + (y-2)^2}$

The distance from point (x, y) to the line $3x - 4y + 9 = 0$ is $\frac{|3x-4y+9|}{\sqrt{3^2+4^2}}$

These distances must be equal. The equation will be

$$\sqrt{(x-3)^2 + (y-2)^2} = \frac{|3x-4y+9|}{\sqrt{3^2+4^2}}$$

After squaring both sides

$$(x-3)^2 + (y-2)^2 = \frac{(3x-4y+9)^2}{25}$$

$$25(x^2 - 6x + 9 + y^2 - 4y + 4) = 9x^2 + 16y^2 - 24xy + 54x - 72y + 81$$

from which

$$16x^2 + 9y^2 + 24xy - 204x - 28y + 244 = 0$$

Answer: $16x^2 + 9y^2 + 24xy - 204x - 28y + 244 = 0$